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Time : 55 minutes
Fall 2017-18

MATHEMATICS 218
QUIZ I

NAME
ID#

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12
2 W	1 W	11 W	2 M	1 M	11 M	4 M	3 M	2 Th	11 F	4 F	5 F

PROBLEM	GRADE
PART I	
1	14 / 14
2	13 / 13
3	9 / 9
4	15 / 15

PART II

5	6	7	8	9	10	11
a	a	a	a	a	a	a
b	b	b	b	b	b	b
c	c	c	c	c	c	c
d	d	d	d	d	d	d
e	e	e	e	e	e	e

5-11 28 / 28

PART III

12	13	14	15	16	17	18
T	T	T	T	T	T	~~T~~
F	F	F	F	F	F	F

12-18 21 / 21

TOTAL 100 / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Find the values of a and b for which the following system

$$\begin{aligned} x + y + z &= b \\ -3x - 3y + az &= 8 \\ 2x + y + z &= 1 \end{aligned}$$

has

a. no solution
b. a unique solution
c. infinitely many solutions.

[14 points]

$$\begin{array}{l} \\ R_2+3R_1 \Rightarrow R_2 \\ R_3-2R_1 \Rightarrow R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & b \\ -3 & -3 & a & 8 \\ 2 & 1 & 1 & 1 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 0 & 0 & a+3 & 8+3b \\ 0 & -1 & -1 & 1-2b \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 0 & -1 & -1 & 1-2b \\ 0 & 0 & a+3 & 8+3b \end{array}\right)$$

a) if $a+3=0$ and $8+3b \neq 0$
$a=-3$ and $-\frac{8}{3} \neq b$
then no solution

b) if $a+3 \neq 0$ $a \neq -3$ then the system has a unique solution

c) if $a+3=0$ and $8+3b=0$
$a=-3$ and $b=-\frac{8}{3}$

$$\begin{array}{cc} \cancel{A}\,B & I \end{array}$$

$$R_1 \leftrightarrow R_3: \quad \left(\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$R_3 - 2R_1 \Rightarrow R_3 \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

every step is written next to the current matrix on which this step is to be performed on

$$\begin{array}{c} \\ \times \frac{1}{3} \\ \times -\frac{1}{3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 & 0 & -2 \end{array}\right)$$

$$R_1 - 2R_3 \Rightarrow R_1 \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} \end{array}\right)$$

$$\begin{array}{cc} I & B^{-1} \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} \end{array}\right)$$

$$(A^{-1} + 2I)^{-1} = \begin{pmatrix} \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & \frac{1}{3} & 0 \\ -\frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix}$$

2. Consider the matrix

$$A=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},$$

(a) Find $(A^{-1}+2I)^{-1}$

[8 points]

Interchange row 1 and 3

$$\begin{array}{c} \quad A \qquad\qquad I \\ \left(\begin{array}{ccc|ccc}0&0&1&1&0&0\\0&1&0&0&1&0\\1&0&0&0&0&1\end{array}\right)\end{array}$$

$$\begin{array}{c} \quad I \qquad\qquad A^{-1} \\ \left(\begin{array}{ccc|ccc}1&0&0&0&0&1\\0&1&0&0&1&0\\0&0&1&1&0&0\end{array}\right)\end{array}$$

by augmenting the matrix A with the identity matrix I and changing A into I ~~=~~ the matrix on the right becomes A which is also A^{-1}

$$B=(A^{-1}+2I)=\begin{pmatrix}2&0&1\\0&3&0\\1&0&2\end{pmatrix}$$

(b) Find A^{11}

[5 points]

$A.A^{-1}=I$ and since $A^{-1}=A$ then

$A^2=I$

$A^{11}=(A^2)^5.A=I^5.A=I.A=A$

3. Show that for all real numbers a, b and c , the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ can be written as a linear combination of the vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

[9 points]

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 2 & 2 & b \\ 0 & 0 & 3 & c \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & 1 & b-a \\ 0 & 0 & 3 & c \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2a-b \\ 0 & 1 & 1 & b-a \\ 0 & 0 & 3 & c \end{array}\right)$$

$$\cancel{\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2a-b \\ 0 & 0 & 1 & b-a-4/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right)}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2a-b \\ 0 & 1 & 0 & b-a-4/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right)$$

so vector $v\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ can always be written of the vectors v_1, v_2, v_3 since the matrix formed by these 3 vectors can be reduced to row echenal form

4. Let A and B be 2×2 matrices such that AB+2A = -I.

(a) Find an expression of A^{-1} in terms of B

[8 points]

$$A(B+2I) = -I$$

$$A\underbrace{\frac{(B+2I)}{-1}}_{A^{-1}} = I$$

$$A^{-1} = \frac{B+2I}{-1} = -B-2I$$

(b) Suppose that the above matrices A and B are given to be A =B= $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$ for some nonzero number a. Find all possible values of a.

[7 points]

$$AB = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^2 & 0 \\ 0 & a^2 \end{pmatrix}$$

$$AB + 2A = \begin{pmatrix} a^2+2a & 0 \\ 0 & a^2+2a \end{pmatrix}$$

$$AB+2A+I=0$$

$$AB+2A+I = \begin{pmatrix} a^2+2a+1 & 0 \\ 0 & a^2+2a+1 \end{pmatrix} = 0$$

then $a^2+2a+1=0$

$(a+1)^2=0$

$a=-1$

PART II. **Circle the correct answer for each of the following multiple choice problems (Problem 5 to Problem 11) IN THE TABLE IN THE FRONT PAGE . [4 points for each correct answer].**

5. *Let A be a* 3×3 *matrix such that* $A^2 = A$. Then,

   a. A is invertible
   b. det (A) = 0
   c. A = I
   d. $A^5 = A^2$
   e. None of the above.

$A^2A^2A = A.A.A = A^2.A$

[4 points]

6. *If A and B are invertible* 3×3 *matrices such that*

$$\det(2A^{-1}) = 2 = \det(A^3(B^{-1})^t)$$,

Then,

   a. det (B)=1/2
   b. det (B)= 2
   c. det (B)= 32
   d. det (B)= 8
   e. None of the above.

$2^3 \det(A^{-1}) = 2$

$2^2 = |A|$

$\det(A^3) . \det(B^{-1}) = 2$

$2^{2^3}/2 = |B|$

[4 points]

7. Which **one** of the following statements is **TRUE**?

   a. If AB = AC, then B= C.
   b. If AB= 0 then A = 0 or B = 0.
   c. If A is invertible then Ax = x has only the trivial solution for x.
   d. If b can be written as a linear combination of the columns of A then AX = b is consistent.

$(A-I)x = 0$

[4 points]

8. Let A be an <u>invertible</u> n×n matrix. Which **one** of the following statements is **<u>FALSE</u>:**

   (a) ~~A^t is invertible.~~
   (b) The number of nonzero rows in a row echelon form of A is n.
   (c) AB is invertible for any n×n matrix B.
   (d) ~~det (A) ≠0.~~
   (e) ~~The reduced row echelon form of A is I.~~

[4 points]

9. Let v_1, v_2, v_3, v_4, and v_5 be vectors in R^n such that $v_3 = v_1 + v_2$, and $v_5 = v_3 + v_4$. Which **one** of the following statements is **FALSE**:

(a) v_5 is a linear combination of v_1, v_2, and v_4
(b) v_2 is a linear combination of v_1 and v_3
(c) v_1 is a linear combination of v_4 and v_5
(d) v_3 is a linear combination of v_4 and v_5
(e) v_3 is a linear combination of v_1 and v_2

[4 points]

10. Let A be an n×n matrix. Which **one** of the following statements is **FALSE**:

(a) If the reduced row echelon form of A is I, then A is invertible
(b) If A is not invertible, then the matrix equation AX=**b** has infinitely many solutions for all **b**.
(c) If the homogeneous matrix equation AX=**0** has only the trivial solution, then A is invertible
(d) If the reduced row echelon form of A is **not** I, then det(A)=0
(e) If A is invertible, then the homogeneous matrix equation AX=**0** has only the trivial solution

[4 points]

11. If $A=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ such that $|A| = 5$, then $\begin{vmatrix} a & c & b \\ 3a+d & 3c+f & 3b+e \\ 2g & 2i & 2h \end{vmatrix}$ is equal to:

(a) -30
(b) 30
(c) 10
(d) -10
(e) None of the above

$$2\begin{vmatrix} a & c & b \\ d & f & e \\ g & i & h \end{vmatrix}$$

$$-2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

[4 points]

PART III. Answer **TRUE** or **FALSE** only, **IN THE TABLE IN THE FRONT PAGE** (**3 points for each correct answer**)

12. If A is any square n×n matrix, then A^tA is symmetric. T $(A^tA)^t = A^tA$

13. Every triangular matrix is invertible. F

14. If A and B are n×n matrices such that AB=0 such that A≠0, then B=0 . F

15. If A and B are n×n matrices such that A is invertible and $(A^{-1}B)$ is invertible, then B is invertible. T

16. The vector $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ is a linear combination of the vectors $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ T

17. If A is a 3×3 matrix such that det $(2A^{-1})$ =4, then det(A^t)=2 T

18. If B is a 2×2 matrix such that $B^T = -B$, then B is not invertible F

[21 points]

$\frac{2^3}{|A|} = 4$

$|A| = 2$

$\begin{matrix} a & b \\ c & d \end{matrix}$

$\begin{matrix} -a & -b \\ -c & -d \end{matrix}$

$\begin{matrix} a & c \\ b & d \end{matrix}$

$c = -b$

$\begin{matrix} 0 & c \\ c & 0 \end{matrix}$

$\begin{matrix} 0 & -c \\ c & 0 \end{matrix}$

$\begin{matrix} 0 & c \\ -c & 0 \end{matrix}$

$0 + c^2 = c^2$